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PRACTICES FOR IDENTIFYING, SUPPORTING AND DEVELOPING MATHEMATICAL GIFTEDNESS IN SCHOOL CHILDREN: THE HUNGARIAN SCENE (LONG VERSION)

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Abstract

The so-called traditional Hungarian mathematics education has multiple roots.

In the mid 19th century as part of the Austro-Hungarian Monarchy, the Hungarian educational system was determined by the Austrian educational program, Entwurf†, which actually relied on the new educational idea that mathematics has to be understood, not just learnt. At the end of the 19th century, Baron Loránd Eötvös, a physicist who became the Minister of Education, had a huge influence on the development of the Hungarian mathematics (and physics) education. In the mid 20th century, mathematics education has gained a new momentum and a new approach emerged.

All through these times, many nationwide mathematics competitions made it possible for children to compare their knowledge. Extracurricular mathematics activities were and still are very popular.

1. *Traditions*

Around the middle of the 18th century, due to the industrial revolution, there was an extreme need of bringing up a new layer of technically skilled workers, as it was recognized in Germany. The idea was acknowledged by the Austrians. This led them to renew the educational system in the form of a new educational law, the so-called Entwurf made in 1849‡. This law laid the foundations of a new educational system, including a technical direction of education for the age group 14–18. One of its most important ideas from our point of view was that learning is an active process that requires the activity of the students.

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†http://epa.oszk.hu/00000/00035/00030/1999-09-mk-Horanszky-150.html.

‡http://epa.oszk.hu/00000/00035/00030/1999-09-mk-Horanszky-150.html.

The program did not immediately spread to Hungary; however, later, in 1868 it was introduced by Baron EÖTVÖS József (in Hungarian, family name comes first followed by given names and we are going to follow this order of names and will capitalize the family names), at that time the Minister of Religion and Education of Hungary. The law did not come into effect until about the 1890s.[§]

The scientific life was the privilege of the nobility but there was a high scientific life. Long before the program of Entwurf (1849) was announced, in 1781 a society called the Hungarian Scientific Society, and in 1824 the Science Popularizing Society was founded in Hungary. The Hungarian Academy of Sciences was founded in 1825. The predecessor of Bolyai Mathematical Society was founded in 1891 under the name Mathematical and Physical Society.

By the end of the 19th century several well-known mathematicians took part in public education, which greatly raised its level, thus, those few who could afford attending school had excellent teachers.[†] Some of the best-known teaching mathematicians of those times were RÁTZ László[‡], the teacher of NEUMANN János (known as John von Neumann) and WIGNER Jenő (known as Eugene Wigner); BEKE Manó[§], a mathematician, member of the Academy of Sciences, teaching at a high school until 1895, who worked on the reforms on mathematics education; MIKOLA Sándor[¶], a physicist, also the teacher of Neumann and Wigner, textbook author; ARANY Dániel, a mathematician, founder of the Mathematical Journal for Secondary School students (age 14–18) including a problem solving contest, KöMaL for short. This journal was actually the second such journal, following the French "*Journal des Mathématiques Élémentaires*", 1877.[‖] The Hungarian journal, which is celebrating its 125th anniversary, was founded in 1893, later edited by RÁTZ László, who made it possible for all students in Hungary, almost independently of their financial circumstances, to take part in the competition and show their interest and talent in mathematics. Apart from a few months during the war years, KöMal has been published ever since. Besides the competition, articles that might have been of interest to the pupils, and teachers as well, were published in the journal. Later the journal was broadened to include physics and, much later, informatics.

Today the problems appear on three different difficulty levels: easy for those who know only as much as taught at school and are about to start to look for challenges

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[§] A. Wieschenberg. The birth of the Eötvös Competition. *The College Mathematics Journal*, 21, no. 4 (1990), 286–293.

[†] B. Szénássy. **History of Mathematics in Hungary until the 20th Century**, Berlin 1992.

[‡] Rátz, László https://en.wikipedia.org/wiki/L%C3%A1szl%C3%B3_R%C3%A1tz.

[§] Beke, Manó https://en.wikipedia.org/wiki/Emanuel_Beke.

[¶] Mikola, Sándor https://hu.wikipedia.org/wiki/Mikola_S%C3%A1ndor.

[‖] http://onlinebooks.library.upenn.edu/webbin/serial?id=jmathelemfr.

1. évfolyam. — Mutatványszám. **Győr**, 1893. December 1-én.

KÖZÉPISKOLAI

MATHEMATIKAI LAPOK.

Megjelenik minden hó 1-én, julius és augusztus hónapok kivételével.

Egyes szám ára . . . . —.25 kr.	**Szerkesztőség:** *Győr, Opitz-utcza 13. szám.*
Előfizetési ár egész évre . . . 2.— frt.	**Kiadóhivatal:** *Győr-Sziget, Hid-utcza 10. szám.*

A lap szellemi részére vonatkozó küldeményeket kérjük a szerkesztőséghez, előfizetési és hirdetési díjakat a kiadóhivatalhoz küldeni.

PROGRAMMUNK.

A **Középiskolai Mathematikai Lapok** első és fő czélja tartalomban gazdag példatárt adni tanárok és tanulók kezébe.

E czél elérésére szolgál az első és negyedik rovat, melyek közül az utóbbi a kitüzött feladatokat, az első ezek megoldásait tartalmazza. Az utóbbi különösen a tanároknak, az előbbi a középiskola felsőbb osztályai ifjuságának szól, a mennyiben a beküldött megoldások közül a legjobb, illetőleg a legjobbak itt közöltetnek.

A második rovat kisebb közleményeket fog tartalmazni, melyek különösen a középiskolai mathematikai tananyag körébe tartozó tételeknek a szokottaktól eltérő és egyszerüsített bizonyításaival, vagy bármely, a mathematikai tanitás methodikájára vonatkozó eszmével foglalkoznak.

Végre a harmadik rovat az iskolai év végével az irásbeli érettségi vizsgálaton feladott tételeket fogja közölni a teljes középiskolák székhelyeinek alfabétikus sorrendjében.

Hasznos szolgálatot vélünk tenni a mathematika tanításának, midőn vállalatunkat, magyar földön ez időszerint az elsőt, megindítjuk. Tankönyveink — kevés kivétellel — nem bővelkednek célszerü, a tanult anyagot kellően megvilágító és e mellett a tanuló-ifjuság érdekét fokozottabb mértékben felköltő példákban. Ez érdeket felkölteni és azt azután fokozni szándékunk, midőn vállalatunknak a folyóirat formáját adjuk. A tanuló várja érdekkel, vajjon beküldött megoldása helyes volt-e és meritsen ösztönzést további buzgó munkálkodásra abból, hogy az ő megoldását találtuk legjobbnak és mint ilyet, közlésre méltónak.

Ezen évfolyamra az 1894. év január 1-étől junius 1-éig az előfizetést ezennel megnyitjuk 1 frt 20 krral.

Minthogy a **Középiskolai Mathematikai Lapok**, közönsége támogatásán kivül, annak közreműködésére is rászorul, tisztelettel kérjük a t. középiskolai szaktanárokat, hogy a **Középiskolai Mathematikai Lapok**-at közleményeikkel felkeresni, és azt, mint a mathematikai tanításnak segédeszközét az ifjúság körében terjeszteni sziveskedjenek.

G y ő r, 1893. november hóban. Kartársi üdvözlettel

A „Középiskolai Mathematikai Lapok"
szerkesztősége.

FIGURE 1. *The first page of the first issue of KöMaL*

in mathematics; moderately difficult for those who can find out more than what is taught at school, who like a challenge, and who do not mind working on a problem for several days; difficult for those who read mathematics beyond what is taught at school and are able to extend their knowledge by themselves. In each problem set there are different levels of problems and the results of pupils from different age groups are evaluated separately for each difficulty level.

Many of our famous mathematicians see this journal as the start of their career.

Just to mention a few of the best known mathematicians who have taken part in the competitions of KöMaL†:

> FEJÉR Lipót; ERDŐS Pál; SZELE Tibor; SURÁNYI János; RÉVÉSZ Pál; LOVÁSZ László (among other prizes, won the Wolf Prize); LACZKOVICH Miklós (the Ostrowski

†http://www.matud.iif.hu/03jul/filep.html.

> Prize); BABAI László (the Gödel Prize, the Knuth Prize); TARDOS Gábor (the Erdős Prize); CSÖRNYEI Marianna (the Philip Leverhulme Prize). SZEGŐ Gábor; HAJÓS György; HÓDI Endre; FUCHS László; CSÁSZÁR Ákos; KÁROLYHÁZY Frigyes.

The first and last sentence of the first page of the first issue of the journal are:

> "The most important and main aim of the journal *Mathematical Journal for Secondary Schools* is to give a rich problem-book to teachers and pupils."

...

> "Since besides the support of the readers, the *Mathematical Journal for Secondary Schools* requires their contribution as well, we sincerely ask the teachers to support the journal with their articles and as a tool of mathematics education spread them among the youth."

To give an idea of the types of the problems the journal posed, we present a few of them here. The first problem discussed in the topic of geometry is:

> *Express the radius of a circle as the function of the four sides of an inscribed quadrilateral.*

One of the first problems of the competition is:

> *Solve the following system of linear equations of first degree:*

$$\begin{aligned} x + \ \ y + \ \ z &= 1, \\ ax + by + cz &= h, \\ a^2x + b^2y + c^2z &= h^2. \end{aligned}$$

Recognizing the importance of the journal, the State of Hungary honoured it with 'Hungarian Heritage Prize' in 2012.

As mentioned earlier, in 1891 the Hungarian Mathematical and Physical Society was founded by EÖTVÖS Loránd, a recognized physicist, son of EÖTVÖS József; and KŐNIG Gyula, a mathematician working in algebra and set theory among other topics. Its aim, as Eötvös said, was to

> "Learn from each other to be able to teach better."

In 1894 the society initiated a competition for 18 years olds, the first in the world. The competition was acknowledged all around the world and the problems were published in the so-called Hungarian problem book†. In ‡ you can read the history of mathematics education in Hungary around the turn of the 19th century. We warmly encourage the Reader to read its 'Preface to the American edition'. The

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†Hungarian Problem Book, see e.g. amazon.com/Hungarian-Problem-Book-Number-English/dp/0883856115 Hungarian Problem Book I. Based on the Eötvös Competitions, 1894–1905 (120 pp.); Hungarian Problem Book II. Based on the Eötvös Competitions, 1906–1928 (128 pp.). J. Kürschák. Revised and edited by G. Hajós, G. Neukomm, and J. Surányi. Translated from the Hungarian revised edition (1955) by E. Rapaport. Random House, New York; Singer, Syracuse, N. Y., 1963.

‡**Hungarian Problem Book**, translation by E. Rapaport, https://es.scribd.com/document/322771120/Hungarian-Problem-Book-I-Number-11-Bk-1-E-Rapaport.

problems of these competitions can be found in [§]. To commemorate KÜRSCHÁK József, the competition was later named after him.

The first problem from 1894:

> *Prove that the expressions $2x + 3y$ and $9x + 5y$ are divisible by 17 for the same set of integral values of x and y.*

(It is a general idea to make the first problem somewhat challenging but easy enough for almost everybody to be able to solve.)

A new generation of young and talented mathematicians grew up for whom new competitions were needed.[†] In 1923 a new nationwide competition was initiated, and is still organized today every year for the age group 16–18. Mathematicians like

> SZEKERES György; KLEIN Eszter; GRÜNWALD Géza; SVÉD György; WACHSBERGER Márta; ERDŐS Pál; TURÁN Pál; GALLAI Tibor

etc. were part of this new generation. In the meantime, as the number of teachers increased, it became necessary to renew the system of teacher training with prescribed curriculum and regulations.[‡]

Until the mid-20th century, the main trends were kept unchanged in mathematics education. Most of the politicians found it important to keep the level of education high. However, education before WWII was not compulsory beyond the age of 14, meaning that only a few percent of the children had access to the higher levels of mathematics education.[§] Not only was the age limit of compulsory education low (14), also not many families could afford schooling. Education was free though not compulsory beyond the age of 14, and families seriously needed the wages of their working children.

2. *New ways in the education system*

After WWII the system of education changed. In the socialist era it was considered to be rather important to lift up poorer children, that is the children of the industrial and agricultural workers, and to support their education. It was very important for the leaders of that time to find more talented children. There were more children of the age group of 14–18 taking part in the education, but the education and the curriculum was not prepared for these changes. Mass education,

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[§] Problems of the oldest mathematical competitions of modern age in Hungary. http://matek.fazekas.hu/images/versenyek/orszagos/kurschak/ek_competitions_18942003.pdf

[†] https://hu.wikipedia.org/wiki/Orsz%C3%A1gos_K%C3%B6z%C3%A9piskolai_Tanulm%C3%A1nyi_Verseny.

[‡] http://www.tanszertar.hu/eken/2009_04/mm_0904.htm.

[§] http://www.mjsz.uni-miskolc.hu/201201/12_kazuskamelinda.pdf.

however, required a more sophisticated policy and curriculum. The school system changed. Previously there was a lower primary education (4 years), then a higher primary school (2–3–4 years) and secondary education (4 years) to prepare for university. From the middle of the 20th century the school system was subdivided into an 8 year primary school and a 4 year secondary school education. In secondary education there was the possibility to specialize.

In the 1950's a new textbook was written for 9th graders by Péter Rózsa and Gallai Tibor, both excellent theoretical mathematicians. Textbooks for further grades involved more or other academic authors, such as Surányi János, who was also the re-launcher of KöMaL after WWII, Tolnai Jenő later the chief editor of KöMal, Hódi Endre, who was the leader of the Hungarian math olympic team for years.

It was difficult to teach mathematics in a class where the mathematical skills of the children varied widely. Teachers had difficulty preparing lessons for groups which included some children who were way ahead, and others way behind the average in the class. Parents also often demanded that their children be given more suitable tasks – different from the rest of the children.

In 1962 a special type of mathematics program started for grades 9–12, which was later widened to grades 7–12. This special program was initiated by Rábai Imre, a teacher in mathematics, following the Soviet model of the same type of classes, but fitted to the Hungarian system of education. The years spent in these classes had a decisive role in the research of many scientists. This form of education is still very popular, however only about 2% of the pupils have the chance to attend such a class.

The above changes affected only pupils of age 14–18. According to the ideas of Varga Tamás, if you do not start teaching mathematics from the very beginning, you might have problems later, when you want to teach more abstract thoughts. This made it necessary to start teaching mathematics in grades 1–8 as well. (Beforehand, the subject was called counting and measuring and was far from mathematics.) This led to a reform that started around the early 1960's and was a long process. The government supported the idea, since this happened to be the era when space programs started, which required a change in education as well.

Formerly, pupils between the age of 6 and 10 had to acquire the basic skills of counting, carrying out basic operations, and learning the rules of these. This tended to emphasise procedural knowledge rather than conceptual knowledge. The reforms initiated by Varga and developed by many practicing teachers aimed at a deeper understanding of mathematics by students, encouraging children to have their thoughts and opinions, raising their intrinsic motivation and making them interested in creating mathematics this way. Beside the idea of teaching "real mathematics" from the very beginning, the most important guiding rules were to let children develop in their own pace, give them time to experiment and think and let them speak up for their own thoughts.

This change in the ideology of teaching mathematics for young children seemed to be a big jump, and many teachers and parents had difficulties adapting to the new ideas. The suggestion of the group of teachers working on the educational research was to introduce a new curriculum to a few schools at a time, and give training to teachers in the schools involved.

In spite of the suggestions and reservations, in 1978 the reform was compulsorily introduced into all schools in Hungary. Though teacher training followed the new guidelines, some older teachers had difficulties acquiring the required knowledge.

The reform started slowly and had its downsides, but all in all it seemed to be successful.

During this research and reform, new competitions were organized for younger children (age 9 to 14), where they met challenging mathematical problems, all requiring some mathematical thinking, beyond what they learnt at school. After their introduction in 1989 the number of competitions increased even more especially for lower age groups (6 to 14).

2.1. *Schools today* Another important component of the Hungarian mathematics education system is the possibility of "leveling". The minimum number of lessons in mathematics in a class is 3 lessons per week, and the maximum is 10. This varies according to the grade, the type of school and the type of program. In today's school system in Hungary it is possible every 2–4 years to change the level at which a child studies mathematics. Children, parents, teachers and other supporting personnel, like psychologists, developers etc. work together to find out if another type of class or school suits a child better than the present one. However, this does not happen very frequently; and rarely in sparsely populated areas. All children in grades 1 to 4 learn approximately the same content, therefore, it is not typical to change at such an early age. Usually, it is the parents who initiate such a change at early ages. Officially, there is a possibility to change after grades 4, 6, and 8, when a child enters high school. Pupils have to sit a test in mathematics (whatever reason they change for) and in Hungarian literature. The majority of the pupils remain at their initial choice. Some pupils change after grade 4, more often after grade 6, and most frequently after grade 8.

In grades 11 and 12 it is very rare that pupils change class or school. However, they have to specialize in a group of subjects taking fewer or more lessons in mathematics. This system makes it possible for children to learn at their own level and according to their interest. During the last two years of their studies (grade 11 and 12), children have to concentrate on their final (and university entering) exam. The final exam in mathematics is compulsory in Hungary. There are levels on the final exam as well. Any good results pupils might have achieved at the national competition count as well when entering university, so it is important for the best ones to participate.

2.2. *Out of school activities* Apart from education at school many children take part in out of school activities. These activities can happen in the afternoons during the school year but some happen during the holidays. The holiday period of children is generally much longer than that of their parents and the parents like to see their children taking part in useful programs during the holidays, like day camps. Some children are invited to such camps according to the results they achieved at competitions, but for most of the children the camp has to be a playful pastime with a lot of physical activity. There is a huge interest in such camps, so the number of mathematics camps is increasing.

2.3. *Teacher training* In the early 1960's, teacher training and training of applied and theoretical mathematicians were separated at university level. The training for prospective teachers for grades 1 to 4 has had almost the same structure for the last one hundred years but the mathematical content was widened to make teachers able to teach according to the new curriculum of primary schools. Almost the same can be said about prospective teachers for grades 9 to 12. However, the training of prospective teachers for grades 5 to 8 has been subject to varying pressures over time. At times it was seen as very important to train them separately and require less knowledge in mathematics (between about 1980 and 2005); at other times it was seen as more important to give them the same degree of mathematical literacy as is given to prospective teachers for grades 9 to 12.

The debate about the best form of teacher training has been going on for a long time and there is still no consensus about it. At the beginning the most important question was whether the level of education for teachers should be as deep as it is for researchers to make them scientifically highly educated teachers, as was often the case in the last decade of the 19th century; and whether they should learn didactics, or better, how much part didactics should take in their education. But having researcher teachers seems to be impossible for two reasons. First of all, many teachers are needed and not that many people are interested in research, and secondly prospective teachers learn at most half as much mathematics as those taking theoretical studies. In Hungary prospective teachers have to take another subject and in one third of their time they learn theoretical educational subjects. Their education includes, among others, a course called 'Elementary mathematics' of 6 semesters, which aims at developing their problem solving skills; and a course called ‘Teaching mathematics’ of 4 semesters which discusses the grasp of teaching certain mathematical topics or managing talented pupils.†

Today, the education of prospective teachers differs significantly according to the two main phases of schooling:

- teachers for children in grades 1 to 4 are prepared for teaching all subjects on a basic level with didactic and psychological background;
- teachers for grades 5 to 12 specialize to teach two subjects and are taught their mathematics subjects at a high level – high enough to carry out mathematical research – with the needed didactic background.

The latter training is subdivided into two levels: one for those who want to teach in grades 5 to 8 only; and one for those who want to teach in grades 5 to 12 (they take two more semesters).

3. *The role of competitions and camps*

Competitions and the interest in them (from both the teachers' and the children's side) gave rise to after school mathematics programs. The most popular competitions are the ones organized in schools. Most of these competitions haven't even got a particular name. The best contestants from such competitions can take part in the competitions of local districts, cities and counties. The winners of the county competitions can take part in nationwide competitions. The best ones may then go to international competitions.

Some competitions can be entered directly, that is, without taking part at school, district or county level competitions, but most competitions have rounds.

The nationwide competitions are organized by different organizations, including the Bolyai Mathematical Society, the Ministry of Education, the Science Popularizing Society and a society founded after the 1989 political opening of the country by groups of private entrepreneurs (e.g. MATEGYE).

After the opening of 1989 entrepreneurs founded new mathematical competitions: while state competitions are financially supported by the state, these competitions had fees. Some of the longest standing entrepreneurships are MATEGYE (founded in 1992), ZALAMAT (founded in 1993), TOLNAMAT (1991). MATEGYE is also the current publisher of a mathematical journal for children aged 6 to 14, that includes a points competition, sometimes referred to as the little sibling of KöMaL, that was originally founded in 1994 by Róka Sándor, a former teacher,

†http://real.mtak.hu/20977/1/Pukanszky_Tanarkepzes_Pecs.pdf, http://mek.oszk.hu/14600/14688/pdf/14688_11.pdf.

university professor. He was the editor in chief as well for several years before he let MATEGYE care for the journal.

3.1. *The best known nation-wide competitions* The 'Kürschák József' competition is the most famous competition, organized by Bolyai Mathematical Society since 1894, for those who are about to finish or have finished their secondary school studies within a year. The competition was originally called 'The Competition of the Mathematics and Physics Society', then in honour of Eötvös Loránd, it was named after him, and after 1949 it took the name Kürschák. The problems require extended answers (in contrast to multiple choice problems). This is the only competition which has only one round. Interestingly, even students entering a university outside Hungary take part in the competition, making it somewhat international. Approximately 0.1 percent of all students of the given age group participate. Participation is free.

The National Competition is for grades 11–12 and it is organized by the State of Hungary in all major subjects. The competition in mathematics started in 1923. The result achieved on this competition counts when entering university. (The best ones don't have to sit the entering exam.) The competition has three levels (categories) and three rounds for categories I and II (for most of the students) and two for category III (for students studying mathematics in special mathematics classes). The problems require extended answers. Approximately 3 percent of all students of the given age group participate. Participation is free.

The 'Arany Dániel' competition, organized by the Bolyai Mathematical Society, is for grades 9–10. The first such competition was organized in 1947. Similarly to the National Mathematics Competition, it has three categories and three rounds for categories I and II and two for category III. The problems require extended answers. Approximately 5 percent of all students of the given age group participate. Participation is free.

The 'Varga Tamás' competition, on behalf of the Bolyai Mathematical Society organized by MATEGYE, is for grades 7–8. The competition started in 1988. It has three rounds and runs in one category (the same level for all students). The problems require extended answers. Approximately 15 percent of all students of the given age group participate. There is a fee for participation.

The 'Zrínyi Ilona' competition, organized by MATEGYE since 1990, for grades 2 to 12, is in the system of multiple choice questions. There are two rounds. Approximately 10 percent of all students of the given age group participate. There is a fee for participation.

The 'Kalmár László' competition, organized by Science Popularizing Society, is for grades 3 to 8 since 1971. There are two rounds and no categories. The problems require extended answers. Approximately 1 percent of all students of the given age group participate. There is a fee for participation.

There are more nationwide competitions but the aforementioned ones are the most popular. Besides these, there are several county competitions in mathematics. These competitions have only one round, but in fact the schools organize local competitions for the children to decide who can go to the county competition. Besides the above mentioned competitions, there are competitions we do not keep track of. It is interesting though that Hungary also takes part in international competitions as well, like Mathematical Kangaroo.

‘Bear events’†. Lately, a new style of out of school math activity came from a group of young people who feel passionately about Mathematics Education. Their activities include outdoor maths challenges, maths camps, experience days and treasure hunts. They all run under the MEDVE brand, which means ‘bear’ in Hungarian. MEDVE refers to/comes from the outdoor venues and the playful atmosphere. In 2017, they reached more than ten thousand pupils and students in Hungary and the figure is rapidly growing every year. They think that their events are excellent platforms for students to meet like-minded people. These friendships and networks can be long-lasting and impactful. Their policy includes using their events and the modern technology to promote a healthy and active lifestyle and that mathematics provides an excellent platform to develop critical thinking and improve problem solving skills. As part of it, they encourage students to think outside of the box in all situations

Their story goes back to 1999 at the Fazekas Mihály Grammar School in Debrecen, Hungary, when the first outdoor maths challenge and maths summer camp was organized for students. These camps and games had such an impact on these students that many of them started working on organising these events after they graduated. In 2012 a group of them took over the baton and the initiative took new momentum. A year later they founded the Mathematics Connects Us Association to be able to work more effectively. Their activity has been growing ever since the beginning. In 2017 they have organised 8 Outdoor Maths Challenges and 6 Maths Camps reaching over 10 000 people; also, they celebrated their 100th event.

3.2. *Mathematics camps* In the late seventies the best students from competitions were gathered together, and mathematical camps were organized for them during school holidays. It was initiated and run privately, and is now run by ‘The Joy of Thinking’ foundation. There was a lot of interest from children, their parents and teachers as well.

† http://medvematek.hu/en/.

Some years later, the Ministry of Education organized a camp for the best students from mathematics competitions as part of their prize. These camps were popular among children and teachers as well. Some invited teachers gave lessons to the children and some other teachers took part as observers. In the last decade, however, there are more and more camps organized and not only for talented pupils but for all pupils interested. Some of them are organized by teachers (most of whom participated in the first camp) and supported by their schools, some by the local council, some by the mathematical journals and some are more popular and better known than others.

Some of the most popular camps are:

- the camp of the foundation 'The Joy of Thinking', supported by the Rényi Mathematical Institute (among others);
- MaMut (MAtheMatical amUsemenT);
- MedveMatek (Bear Math)[†].

This wide selection of competitions and the different levels of mathematics education makes it possible for every pupil to reach his or her best result. This way talented pupils are supported within the system.

This emphasis on encouraging mathematics through problem solving seems to be very successful. The variety of competitions and camps make it possible to keep track of the work of the most talented children, keep up their interest and encourage them to develop their skills by preparing for these competitions.

True, not all talented students are interested in competing, yet, most Hungarian mathematics educators are convinced that due to the variety of competitions and other programs, they play an important role in identifying, supporting and developing mathematical giftedness in school children in Hungary.

Talented pupils are constantly monitored by their teachers and regularly sent by them to competitions.

4. *Sample problems from mathematics competitions on different levels for different age groups*

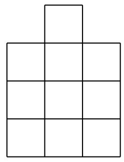
In general, problems of mathematics competitions suit the knowledge of the age group they are designed for. Contestants do not need to know more than what they learnt at school, however the ideas needed to solve these problems are more complex, or one might need an excellent idea to solve them.

Here we try to illustrate what kind of problems pupils meet on competitions.

[†]http://www.mategye.hu/download/cserepek/cserepek.pdf. (In Hungarian.)

4.1. A sample problem for grade 3 students from the county round of 'Kalmár László' competition:

Subdivide the figure



into two pieces so that each contains 5 *grid squares. Color in one of the two parts. Find multiple solutions. (Two solutions are considered identical if the shapes you get by subdividing are alike in the two solutions.)*

Note to the solution: Take systematic partitions. The solution requires combinatorial thinking.

4.2. A problem from 'Arany Dániel' competition for grade 9 students in 'special mathematics program', final round:

Let k denote the circumscribed circle of the triangle ABC. Denote the bisector point of the arc BC not including A by D; the bisector point of the arc CA not including B by E; the bisector point of the arc AC not including C by F. The inscribed circle of the triangle ABC touches the sides BC, CA and AB at points K, L, M, respectively. Prove that lines DK, EL and FM meet in one point.

Note to the solution: Any two circles in the plane are similar.

4.3. A problem from 'Arany Dániel' competition for grade 10 students in 'normal program', final round:

Prove that we can omit two elements from the 2018*-element set*

$$H = \{\, 1!, , 2!,\ 3!,\ \ldots,\ 2017!,\ 2018!\, \}$$

so that the product of the remaining 2016 *elements is a square number.*

Note to the solution: Prime factorization is a key idea.

4.4. A sample problem from Zrínyi competition, first round, "level 3" problems (most difficult) for grades 9–10:

At least how many numbers have to be omitted from the set $\{1, 2, 3, \ldots, 16\}$ *so that neither sum of any two remaining numbers is a square?*
(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

Note to the solution: Consider the square numbers as the sum of two numbers systematically.

4.5. 'Kürschák József' competition, 2008:

In a country there are train and bus connections between cities. There might be both types of connections between some cities but it might happen that one goes one way *and the other one goes the other way. We know that picking any two cities you can get from one of them to the other using only one type of transport (you might have to change, though) but not necessarily the other way. Prove that there is at least one city from which you can travel to all other cities using one type of transport (but the type may be different for different cities).*

Note to the solution: The problem is based on graph theory.

4.6. A sample problem from the journal of KöMaL, moderately difficult, 2018:

Let n be a positive integer. Solve the following system of equations on the set of real numbers.

$$\begin{gathered} a_1^2 + a_1 - 1 = a_2 \\ a_2^2 + a_2 - 1 = a_3 \\ \vdots \\ a_n^2 + a_n - 1 = a_1 \end{gathered}$$

Note to the solution: If you have difficulties solving it for arbitrary n, try $n = 1, 2, 3, \ldots$ first.

4.7. A sample problem from the journal of Abacus, 2004:

There are several routes from Lower Winterslow, Upper Winterslow and Middle Winterslow. We know that the number of direct routes between any two of these three villages is between 3 and 10. We can go from Lower Winterslow to Upper Winterslow directly and also through Middle Winterslow; the number of these routes is 33. Similarly, we can go from Middle Winterslow to U pper Winterslow directly and also through Lower Winterslow, all together on 23 different routes. How many different routes are there between Middle Winterslow and Lower Winterslow (directly and through Upper Winterslow)?

Note to the solution: Combinatorial thinking helps.

4.8. On the 1947 Kürschák competition the following problem was posed:

Prove that in a group of six people either there are three mutual acquaintances or there are three mutual strangers.

Or, as translated in English[†]:

Show that any graph with 6 points has a triangle or three points which are not joined to each other.

[†] **Problems of the oldest mathematical competitions of modern age in Hungary**. http://matek.fazekas.hu/images/versenyek/orszagos/kurschak/ek_competitions_18942003.pdf.

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